STA 6934
Problem 1.11

(Solution prepared by Vladimir L. Boginski**)**

Weibull distribution $We(\alpha, \beta)$:

The density is given by:

$$f(x|\alpha,\beta) = \frac{\beta}{\alpha}\left(\frac{x}{\alpha}\right)^{\beta-1} e^{-\left(\frac{x}{\alpha}\right)^{\beta}},$$

for $x \in \mathbb{R}^{+}$, $\alpha > 0$, $\beta > 0$.

The **cdf** can be expressed as follows:

$$F(x) = \int_0^x f(t|\alpha,\beta)dt = \int_0^x \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1} e^{-\left(\frac{t}{\alpha}\right)^{\beta}} dt = \int_0^x e^{-\left(\frac{t}{\alpha}\right)^{\beta}} d\left(\left(\frac{t}{\alpha}\right)^{\beta}\right) =$$

$$= -e^{-\left(\frac{t}{\alpha}\right)^{\beta}}\Bigg|_0^x = 1 - e^{-\left(\frac{x}{\alpha}\right)^{\beta}}.$$

So we see that the cdf of the Weibull distribution can be written explicitly.

The **hazard rate**:

$$h(t) = \frac{f(t)}{1-F(t)} = \frac{\frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1} e^{-\left(\frac{t}{\alpha}\right)^{\beta}}}{1-(1-e^{-\left(\frac{t}{\alpha}\right)^{\beta}})} = \frac{\beta}{\alpha}\left(\frac{t}{\alpha}\right)^{\beta-1}.$$

It can be seen that the scale parameter α determines the behavior of the hazard rate: the function *h(t)* grows faster (w.r.t. *t*) for smaller values of α, and it grows slower for bigger values of α.