Monte Carlo Statistical Methods

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Problem 3.10

We know that

$$\frac{1}{M}\sum_{m=1}^{M} h(X_m) - - - > \mathrm{E}(h(X))$$

We have to show:

$$E(h(X)) \quad = \quad \frac{\sum_{i=1}^{M} h(y_i)\frac{f(y_i)}{g(y_i)}}{\sum_{j=1}^{M}\frac{f(y_j)}{g(y_j)}}$$

By the given algorithm, we know that

$$X_m = Y_i \text{ with prob } w_i \text{ for all } m=1,\ldots M$$

Now,

$$\begin{aligned} LHS &= E(h(X)) \\ &= \sum_{i=1}^{M} h(y_i) \times w_i \\ &= \frac{\sum_{i=1}^{M} h(y_i)\frac{f(y_i)}{g(y_i)}}{\sum_{j=1}^{M}\frac{f(y_j)}{g(y_j)}} \\ &= RHS \end{aligned}$$