Problem 3.8

STA 6934

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1. $g_1(x) = \frac{1}{2}e^{-|x|}$

$$M = \sup_y \frac{e^{-\sqrt{y}}[\sin(y)]^2}{\frac{1}{2}e^{-|y|}} = \sup_y 2[\sin(y)]^2 e^{-\sqrt{y}+|y|}$$

.

Since $\sqrt{y} < |y|$ for $y > 1$, $-\sqrt{y} + |y| > 0$ for $y > 1$. Thus $e^{-\sqrt{y}+|y|} \to \infty$ as $y \to \infty$ and $M = \infty$. Thus AR no longer apply, an alternative MCMC methods.

2. $g_2(x) = \frac{1}{2\sqrt{2}}\text{sech}^2(\frac{x}{\sqrt{2}})$

$$\begin{aligned} M &= \sup_y \frac{e^{-\sqrt{y}}[\sin(y)]^2}{\frac{1}{2\sqrt{2}}\text{sech}^2(\frac{y}{\sqrt{2}})} = \sup_y 2\sqrt{2}[\sin(y)]^2 e^{-\sqrt{y}} \left(\frac{e^{\frac{y}{\sqrt{2}}} + e^{\frac{-y}{\sqrt{2}}}}{4} \right) \\ &< \sup_y \sqrt{2}[\sin(y)]^2 e^{-\sqrt{y}} e^{\frac{y}{\sqrt{2}}} \end{aligned}$$

Since $\sqrt{y} < \frac{y}{\sqrt{2}}$ for $y > 2$, $-\sqrt{y} + \frac{x}{\sqrt{2}} > 0$ for $y > 2$. Thus $e^{-\sqrt{y}+(y/\sqrt{2})} \to \infty$ as $y \to \infty$ and $M = \infty$. Thus AR no longer apply, an alternative MCMC methods.

3. $g_3(x) = \frac{1}{2\pi}\frac{1}{1+x^2/4}$

$$M = \sup_y \frac{e^{-\sqrt{y}}[\sin(y)]^2}{\frac{1}{2\pi}\frac{1}{1+y^2/4}} = 7.456449$$

.

It occurs at $y = 14.14$ (Figure 1). For one simulation the mean was 6.4835 and the histogram is shown in Figure 2. The Matlab code for the Accept Reject Algorithm is the following:

```
%This program will generate nsim random variables from
%the density f(x)=exp{-sqrt(x)}*[sin(x)]^2 using the
%Accept Reject Algorithm with candidate density Cauchy(0,2).

nsim = 100000;
M = 7.456449;
randvars = [];  %vector of simulated random variables

i = 0;
while (i <= nsim)
   u_1 = rand(1,1);
   u_2 = rand(1,1);
   Y = 2*tan((u_2-1/2)*pi);  %generate Cauchy(0,2)
   if (Y > 0) %consider Cauchy variables such that 0<Y<\infty
     temp = (1/M)*(exp(-sqrt(Y))*[sin(Y)]^2)/(1/(2*pi*(1+Y^2/4)));
     if (u_1 < temp)
         randvars = [randvars;Y];
         i = i + 1;
     end
   end
end
```

4. $g_4(x) = \frac{1}{2\pi}e^{\frac{-x^2}{2}}$ (Corrected by George Casella)

$$M = \sup_y \frac{e^{-\sqrt{y}}[\sin(y)]^2}{\frac{1}{2\pi}e^{\frac{-y^2}{2}}} = \sup_y 2\pi[\sin(y)]^2 e^{-\sqrt{y}+y^2/2}.$$

Since $\sqrt{y} < \frac{y^2}{2}$ for $y > 4^{1/3}$, $-\sqrt{y} + \frac{y^2}{2} > 0$ for $y > 4^{1/3}$. Thus $e^{-\sqrt{y}+y^2/2} \to \infty$ as $y \to \infty$ and $M = \infty$. Thus AR no longer apply.