STA6934. PROBLEM 4.9

Given the transition matrix

$$\text{states:} \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$$

$$\mathbb{P} = \begin{pmatrix} 0.0 & 0.4 & 0.6 & 0.0 & 0.0 \\ 0.65 & 0.0 & 0.35 & 0.0 & 0.0 \\ 0.32 & 0.68 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.12 & 0.88 \\ 0.0 & 0.0 & 0.0 & 0.56 & 0.44 \end{pmatrix},$$

examine whether the corresponding chain is irreducible and aperiodic.

Solution: Consider a graphic representation of the chain:

- the nodes correspond to the states 1,2,3,4 and 5;
- the link numbers correspond to the probabilities of the corresponding transition.

As we see, the chain is splitted into two disjoint chains, so for every state there is an unreachable state in the chain. So, **the chain is not irreducible**. Since there are the same-stage transitions($4 \mapsto 4$ and $5 \mapsto 5$), **the chain is aperiodic**.

Note: The chain consists of two irreducible chains 1-2-3 and 4-5. These subchains are aperiodic, because the period is equal to 1 for both of them.