STA6934 Problem 5.22abc Solution by Joseph Powers

(a) Be warned, I'm of the opinion that there are several typos in this problem. Clearly, $\{X_i\}$ is an iid sequence since, for a fixed value of i, X_i depends only on Z_i, and $\{Z_i\}$ is iid. The iidness of the Z's also means that the success probability will be constant. Hence, the X's really are Bernoulli random variables. Below, let θ be the vector $(\zeta,\ \sigma^2)$. First compute the success probability:

$$P(X_i = 1 \mid \theta) = P(Z_i > u) = P(z > \frac{u-\zeta}{\sigma}) = \Phi(\frac{u-\zeta}{\sigma}) = p \quad (1)$$

(b)Consider $\{z_i\}$ to be the complete date (since given the z's the x's are redundant). The computation of the complete data likelihood is then easy, thanks to independent normality.

$$L^c(\theta \mid \underline{z}) = \prod_{i=1}^{n} f(Z_i \mid \theta) = \prod_{1}^{n} \frac{1}{2\pi\sigma^2} e^{\frac{-(Z_i-\zeta)^2}{2\sigma^2}} \quad (2)$$

$$ln(L^c(\theta \mid z) = -\frac{n}{2} ln(2\pi\sigma^2) - \frac{1}{2\sigma^2} \sum_{i=1}^{n} (Z_i-\zeta)^2 \quad (3)$$

$$Q(\theta \mid \theta_0, \underline{x}) = \mathbb{E}_{\theta_0}\{lnL^c(\theta \mid \underline{x}, \underline{z}) \mid \theta_0, \underline{x}\} = -\frac{n}{2} ln(2\pi\sigma^2) - \frac{1}{2\sigma^2} \sum_{i=1}^{n} \mathbb{E}\{(Z_i-\zeta)^2 \mid \theta_0, \underline{x}\} \quad (4)$$

(c) Remember that the EM sequence is generated by maximizing the previous expectation. So substitute $\hat{\zeta_{(j)}}$ and $\hat{\sigma}^2_{(j)}$ into θ_0 , and then the MLEs are the conditional expectations given on page 224.