**Problem 7.26 Monte Carlo Statistical Methods      by David Finlay**

In the set up of Example 7.2.4, the posterior distribution of N can be evaluated by recursion.

(a) Show that $\pi(N/n_0, n_t) \propto \frac{(N-n_0)!\lambda^N}{(N+1)!(N-n_t)!}$ (Note correction. Also, $n_0$=n, $n_t$=n'.)

**Solution**: From joint posterior of N and p, π(p, N /n, n') = π(p, N/D), D = data=($n_0$, $n_t$) found on page 307, Example 7.2.4, MCSM, by R & C, π(N/D) is found by integrating out p over its support 0 < p< 1.

So $\pi(N/n_0, n_t) \propto \frac{\Gamma(n_0+1)\Gamma(N-n_0+1)\, e^{-\lambda}\lambda^N}{\Gamma(N+2)(N-n_t)!} \propto \frac{(N-n_0)!\lambda^N}{(N+1)!(N-n_t)!}$

(b) Let $C_N = \pi(N/n_0,n_t) / \pi(N-1/n_0,n_t)$. So $C_N = \frac{(N-n_0)\lambda}{(N+1)(N-n_t)}$

$E^{\pi}[N|\, n_0,n_t] = \sum_{N=n_0}^{\infty} N\pi(N/n_0,n_t)$

But $\pi(N|n_0,n_t) = C_N\pi(N-1) = C_N C_{N-1}\pi(N-2) = (\prod_{i=n_0+1}^{N} C_i)\pi(n_0)$

So $E[N|n_0,n_t] = \sum_{N=n_0+1}^{\infty} N\left(\prod_{i=n_0+1}^{N} C_N\right)\pi(n_0 \setminus n_t) + n_0\, \pi(n_0 \setminus n_t)$

(c) The algorithm for the recursion method is:

Set $n_0$=112, $n_t$=79, λ=500, E(N) =0
Set N = $n_0$
Do while $E[N]-E[N-1] \geq \in$;
Compute $\pi(N \setminus n_0, n_t)$
Add $N\pi(N \setminus n_0,n_t)$ toE[N]
Increment N
Compute $C_N$
Compute $\pi(N) = C_N\, \pi(N-1)$
PRINT E[N]

Using the Gibbs−Sampler with : $(N-n_t)\setminus p,n_0,n_t \sim \text{Poisson}(\lambda)$ and $p\setminus N,n_0,n_t \sim Be(n_0+1,N-n_0+1)$

Algorithm: Given $p^{(t)}$,
Simulate $(N-n_t)^{(t)} \sim \text{Poisson}(\lambda, p^{(t)})$
Add $n_t$ to $(N-n_t)^{(t)}$ to get $N^{(t)}$
Simulate $p^{(t+1)} \sim Be(n_0+1, N-n_0+1)$
Evaluate $E[N \backslash D] = \sum_{t=1}^{k} N^{(t)} / k$