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STA 6934 – Homework 2
Problem 2.1

**(a)** $f(x) = \frac{1}{\sqrt{\pi x(1-x)}}$

Let $Y = 1 - X \Rightarrow X = 1 - Y$.

$\frac{dx}{dy} = -1 \Rightarrow \mid \frac{dx}{dy} \mid = 1$.

$f(y) = \frac{1}{\sqrt{\pi(1-y)y}}(1)$

$= \frac{1}{\sqrt{\pi y(1-y)}}$.

Hence the arcsine distribution is invariant, since $f(x) = f(y)$.

**(b)** $X \sim U[0, 1]$

**(1)** For $0 \leq X \leq 1/2$,
$Y = 2X \Rightarrow$
$F_Y(y) = P(Y \leq y) = P(2X \leq y) = P(X \leq y/2) = F_X(y/2) = y/2$.

$X \in [0, 1/2] \Rightarrow 2X = Y \in [0, 1]$

$\Rightarrow f_Y(y) = \frac{1}{2} I_{[0,1]}(y)$

**(2)** For $1/2 \leq X \leq 1$,
$Y = 2(1 - X) \Rightarrow$
$F_Y(y) = P(Y \leq y) = P(2(1-X) \leq y) = P(1 - X \leq y/2) = P(X \geq 1 - y/2) = 1 - F_X(1 - y/2) = 1 - 1 + y/2 = y/2$.

$X \in [1/2, 1] \Rightarrow 2(1 - X) = Y \in [0, 1]$

$\Rightarrow f_Y(y) = \frac{1}{2} I_{[0,1]}(y)$

Combining **(1)** and **(2)**, we get: $f_Y(y) = \frac{1}{2} I_{[0,1]}(y) + \frac{1}{2} I_{[0,1]}(y)$
$\Rightarrow f_Y(y) = I_{[0,1]}(y)$.
Hence the uniform distribution is invariant under the tent transformation, since $f(x) = f(y)$.