Homework II Bernhard Klingenberg

Problem 2.18

1. Proof:

$$\begin{aligned} P(Y < y) &= P(U < y | U < \alpha) = \frac{P(U < y, U < \alpha)}{P(U < \alpha)} \\ &= \begin{cases} \frac{P(U<y)}{\alpha} & \text{if } y < \alpha \\ \\ \frac{P(U<\alpha)}{\alpha} & \text{if } \alpha < y \end{cases} \\ &= y/\alpha \; \mathrm{I}_{(0,\alpha)}(y) \; + \; \mathrm{I}_{(\alpha,\infty)}(y) \end{aligned}$$

   This is the c.d.f. of a $U(0, \alpha)$.

2. Based on the histograms on page 2, there does not seem to be much difference between the two algorithms in generating random numbers, even when α is close to 1.

3. R-code for algorithm A.14:

```
for(a in c(0.8, 0.9, 0.99))
{
n< −12000 s< −10000
x< −runif(n)
x< −x[x<a]
x< −x[1:s]
y< −a*runif(s)
hist(x,main=paste(a),breaks=seq(0,a,length=30),freq=F)
hist(y,main=paste(a),breaks=seq(0,a,length=30),freq=F)
}
```

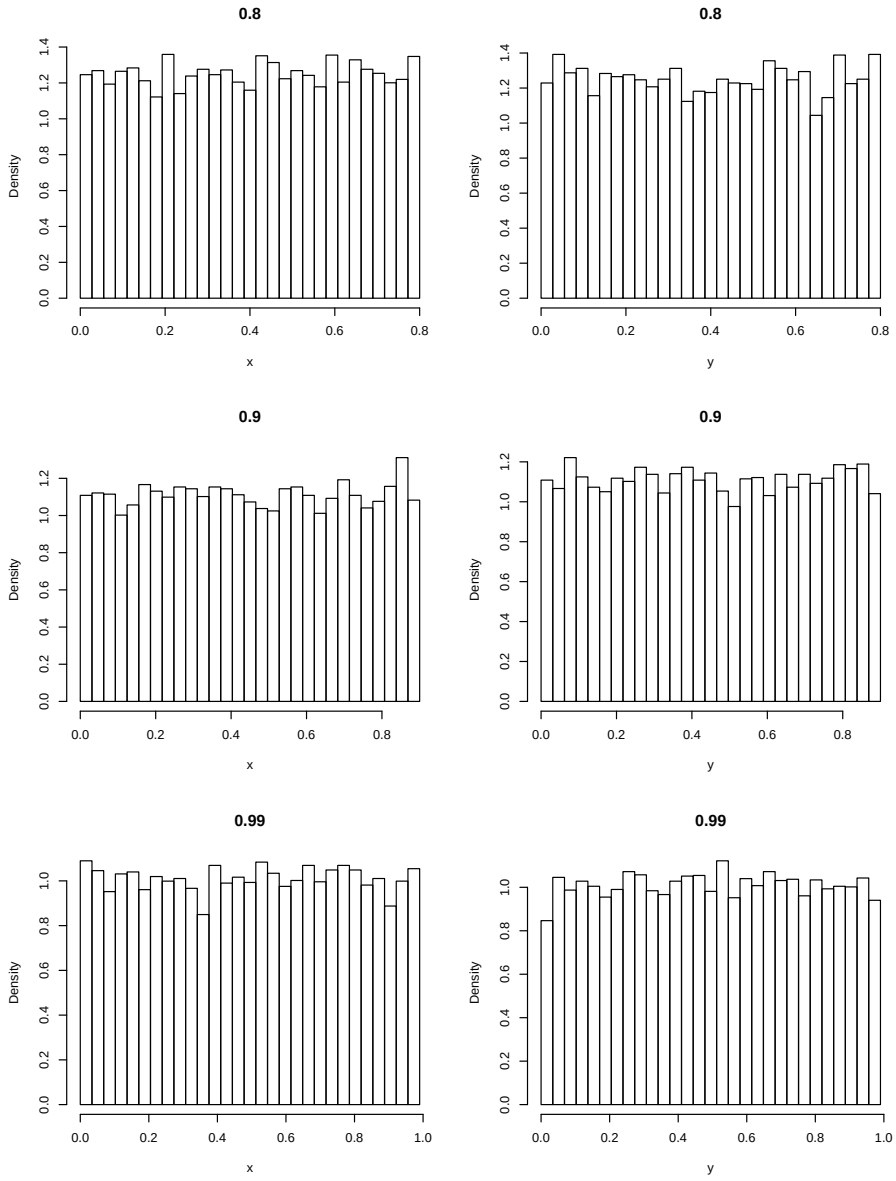

Figure 1: Histograms of 10,000 Uniform(0,α) generated with algorithm A.14 (left column) and the genuine transformation α*U(0.1) (right column).