MCSM
Assignment 2
Problem 2.28a
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$h_1(x|y) \sim N(0, \frac{\nu}{y})$

$h_1(x|y) = \frac{e^{\frac{-x^2}{2(\frac{\nu}{y})^2}}}{\sqrt{2\pi\frac{\nu}{y}}}$

$h_2(y) \sim {\chi_\nu}^2$

$h_2(y) = \frac{y^{\frac{\nu}{2}-1}e^{\frac{-y}{2}}}{\Gamma(\frac{\nu}{2})2^{\frac{\nu}{2}}}$

Now,

$\int_y h_1(x|y)h_2(y))dy = \frac{\int_0^\infty e^{\frac{-x^2y^2}{2\nu^2}-\frac{y}{2}}y^{\frac{\nu}{2}}dy}{\sqrt{2\pi}2^{\frac{\nu}{2}}\nu\Gamma(\frac{\nu}{2})}$

$= \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})\sqrt{\nu\pi}(1+\frac{x^2}{\nu})^{\frac{\nu+1}{2}}}$