Problem 7.9

Compare the usual demarginalization of the Student's *t* distribution discussed in Example 7.1.6 with an alternative using a slice sampler. Produce a table like Table 7.1.1.
(Hint: Use two uniform dummy variables)

Solution
As mentioned in the Example 2.2.6, Student's *t* density with *v* degrees of freedom can be decomposed into a mixture of normal and chi squared distributions:

$$X \mid y \sim N\left(0, {}^{v}\!/_{y}\right),\ Y \sim \chi^2_v .$$

The following R-program uses this representation to generate random numbers distributed according to Student's *t* distribution.

```
v <- 2;
nsim <- 10000000;
memory.limit(1000);
{
    x<-array(0,dim=c(nsim,1));
    for (i in 1:nsim)
    {
            w<-rchisq(1,df=v);
            x[i]<-rnorm(1,mean=0,sd=v/w);
    }
}
```

In the following table we present convergence of the estimated quantiles, based on a Markov chain simulated by the demarginalization of Student's *t* distribution:

n	0	0.816497	1.06066	1.885618	2.919986	6.964557	9.924843	22.32712	70.70007
100	0.48	0.68	0.73	0.88	0.88	0.96	0.98	1	1
1000	0.495	0.734	0.778	0.851	0.894	0.949	0.964	0.984	0.997
10000	0.4951	0.7298	0.7644	0.8404	0.8881	0.9458	0.9615	0.9816	0.9939
100000	0.50065	0.73328	0.76956	0.84467	0.88908	0.9485	0.96298	0.98285	0.99442
1000000	0.49911	0.732757	0.768704	0.841851	0.887352	0.947056	0.961953	0.982643	0.994335
10000000	0.500217	0.733656	0.769787	0.84286	0.88801	0.947505	0.962204	0.982643	0.994413
TRUE	0.5	0.75	0.8	0.9	0.95	0.99	0.995	0.999	0.9999

For the Student's *t* density, a slice generator $f(x) \propto \left(1+\frac{x^2}{v}\right)^{-\frac{1+v}{2}}$ is associated with the two conditional distributions:

$$w \mid x \sim U\left[0, \left(1+\frac{x^2}{v}\right)^{-\frac{1+v}{2}}\right],$$

$$X \mid w \sim U\left[-\sqrt{v\left(w^{-\frac{2}{1+v}}-1\right)}, \sqrt{v\left(w^{-\frac{2}{1+v}}-1\right)}\right].$$

The following R-program implements this generator:

```
v <- 2;
fx <- function(x)(1+x*x/v)^(-(1+v)/2);
invfx <- function(w)sqrt(v*(w^(-2/(1+v))-1));
nsim <- 10000000;
{
        x<-array(0,dim=c(nsim,1));
        x[1]<--log(runif(1));
        for (i in 2:nsim)
        {
            w<-runif(1,min=0,max=fx(x[i-1]));
            x[i]<-runif(1,min=-invfx(w),max=invfx(w));
        }
}
```

In the following table we present the convergence of the estimated quantiles, based on a Markov chain simulated by the slice generator:

n	0	0.816497	1.06066	1.885618	2.919986	6.964557	9.924843	22.32712	70.70007
100	0.43	0.75	0.8	0.87	0.92	1	1	1	1
1000	0.505	0.773	0.821	0.918	0.963	0.999	1	1	1
10000	0.5	0.7515	0.8033	0.9077	0.9556	0.9923	0.996	0.999	1
100000	0.49897	0.75047	0.80086	0.90038	0.95103	0.99051	0.99535	0.99908	0.99995
1000000	0.500429	0.749965	0.800193	0.90021	0.950117	0.989995	0.99501	0.99903	0.999919
10000000	0.499985	0.749872	0.799954	0.900054	0.950141	0.989987	0.994982	0.998975	0.999897
TRUE	0.5	0.75	0.8	0.9	0.95	0.99	0.995	0.999	0.9999

As we see, the slice sampler demonstrates good performance and converges relatively fast. On the other hand, the convergence of the demarginalization sampler is not so evident questionable.